

Lecture 8 on Discrete-Time Convolution, Part 1

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By TA Alice Liu

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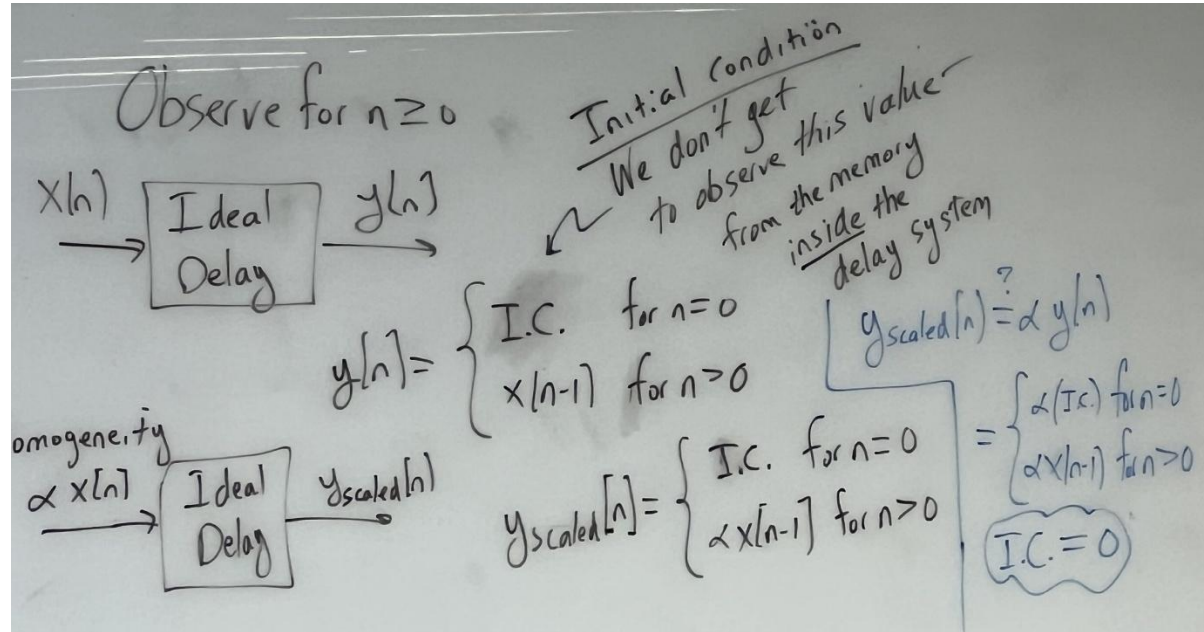
System Properties

- Given a system under observation for all time $-\infty < n < \infty$
 - o We have the input, $x[n]$ (for all time)
 - o We have the output, $y[n]$ (for all time)
 - o Input signal $x[n]$ will be transformed by system T to create output signal $y[n]$
- Now: we want to characterize the system T using its properties
 - o System properties tells us which mathematical tools can be used in analysis
 - o And then we can further characterize the system's behavior
- Time Invariance
 - o If the input is shifted by a certain amount, n_0 , then the output is shifted by the SAME amount for all possible finite integer values for n_0
 - o For the system $y[n] = n x[n]$
 - NOT time-invariant
 - Because little n (part of the SYSTEM T) does NOT shift when we shift the input signal $x[n]$
- Linearity
 - o Linearity must satisfy BOTH
 - Homogeneity
 - Additivity
 - o Additivity
 - If you add 2 inputs and pass them through the system, the resulting output is the addition of their corresponding individual outputs
 - o Homogeneity
 - If we scale the input signal by a constant a , then the output signal should also be scaled by constant a
- LTI systems .. why?
 - o Abstraction of how a system is implemented
 - o Nice analysis tools in the time and frequency domains
 - o Output in the time domain is always the convolution of the input signal and the impulse response

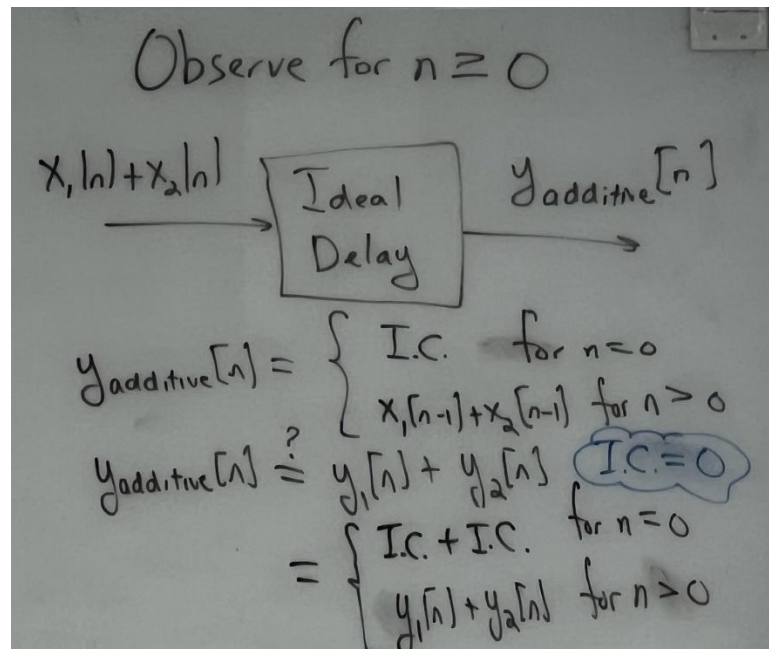
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Initial Conditions

For Homogeneity to hold true, the Initial Conditions must be 0. (necessary condition)



For Additivity to hold true, the Initial Conditions must be 0. (necessary condition)



For Time-Invariance to hold true, it is complicated (please see [Handout U](#)). Setting the initial condition to zero is a special case of the necessary conditions for time-invariance to hold.

Necessary vs Sufficient?

- Necessary -> *that* condition must hold
- Sufficient -> *that* necessary condition alone is not enough (you may need additional conditions that must be satisfied)

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FIR Filter (represented as a convolution) when observing for $n \geq 0$:

Observe for $n \geq 0$

$x[n] \rightarrow \boxed{\text{FIR Filter}} \rightarrow y[n] = \sum_{k=0}^M h[k]x[n-k]$

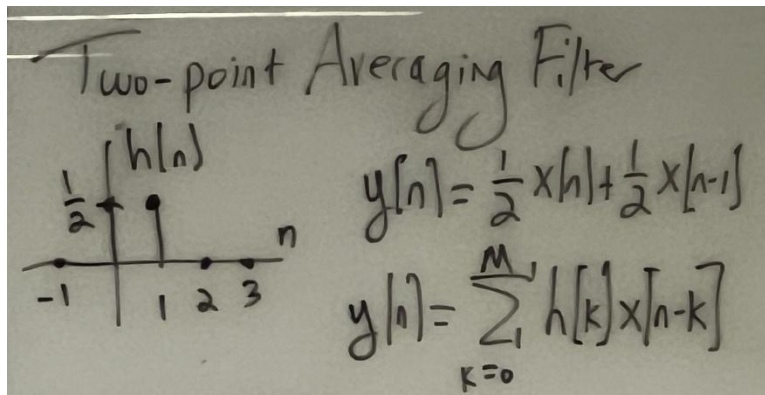
First input value is $x[0]$ when $n=0$

First output value?

$$y[0] = h[0]x[0] + \underbrace{h[1]x[-1]}_{\text{I.C.}} + \underbrace{h[2]x[-2]}_{\text{I.C.}} + \dots + \underbrace{h[M]x[-M]}_{\text{I.C.}}$$

M initial conditions:
Set them to zero as necessary condition for LTI to hold.

FIR Filter Convolution Demonstration using a two-point FIR averaging filter



Demonstration uses the `dconvdemo` function available for download in the [Discrete-Time Convolution Demo](#) from the DSP First Website.